

Prova scritta del 27. 03. 2008 (fila 1)

c1) $X \sim N(\mu, \sigma^2=4)$ $\mu: P[X > 3.6] = 0.0778$
 $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$

$$P\left[Z > \frac{3.6-\mu}{2}\right] = P[Z > z_\alpha] = 0.0778$$

$$P[Z \leq z_\alpha] = 1 - 0.0778 = 0.9222 \Rightarrow z_\alpha = 1.42$$

$$\Rightarrow \frac{3.6-\mu}{2} = 1.42 \Rightarrow \underline{\mu = 0.76}$$

c2) $p = 0.0002 = 2 \cdot 10^{-4}$ $P(X \geq 2)$ in 20 anni

in 20 anni ci sono $52 \cdot 20 = 1040$ settimane $\Rightarrow m = 1040$

$X \sim B(m, p)$ ma $mp = 1040 \cdot 2 \cdot 10^{-4} = 0.208 \ll$

$\Rightarrow B$ può essere approssimata da $P(\lambda = mp)$

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x=0, 1, \dots$$

$$\begin{aligned} \underline{P(X \geq 2)} &= 1 - P(X < 2) = 1 - P(X=0) - P(X=1) \\ &= 1 - e^{-\lambda} - \lambda e^{-\lambda} \\ &= 1 - e^{-0.208} (0.208 + 1) = \underline{0.01885} \end{aligned}$$

si può ragionare anche sui giorni

$$\bar{m} = 20 \cdot 365 = 7300 \quad 7300 : 7 \approx 1043 \text{ settimane}$$

e in questo caso $mp = 0.2086$

c3) $P[X > \sqrt{e} \mid -1 < X < e^3]$ = $\frac{P[X > \sqrt{e} \cap -1 < X < e^3]}{P[-1 < X < e^3]}$

ma $-1 < \sqrt{e} < e^3$

$$= \frac{P[\sqrt{e} < X < e^3]}{P[-1 < X < e^3]}$$

$$= \frac{F(e^3) - F(\sqrt{e})}{F(e^3) - F(-1)}$$

$$= \frac{1 - \ln \sqrt{e}}{1 - 0} = 1 - \frac{1}{2} \ln e = \underline{\underline{\frac{1}{2}}}$$

$$c4) T \sim \text{Unif}(15^\circ\text{C}, 20^\circ\text{C}) \Rightarrow f_T = \begin{cases} \frac{1}{5} & \text{se } 15 < T < 20 \\ 0 & \text{altrove} \end{cases}$$

$$P_1 = P[X > 18^\circ\text{C}] = \frac{1}{5} (20 - 18) = \frac{2}{5}$$

$$P[X \leq 18^\circ\text{C}] = \frac{3}{5}$$

se in 4gg non $T > 18^\circ\text{C}$ in 3gg non $T \leq 18^\circ\text{C}$

$$Y \sim B(m=7, p=\frac{3}{5})$$

$$P[Y=3] = \binom{7}{3} \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^4 = \frac{7!}{3!4!} \cdot \frac{3^3}{5^3} \cdot \frac{2^4}{5^4} = \frac{7 \cdot 3^2 \cdot 2^4}{5^6}$$

Q7)

$$A \text{ indep da } B \rightarrow P(A \cap B) = P(A) \cdot P(B)$$

$$A \text{ indep da } C \rightarrow P(A \cap C) = P(A) \cdot P(C)$$

$$B, C \text{ incompatibili} \rightarrow P(B \cap C) = 0$$

$$\begin{aligned} P[A | B \cup C] &= \frac{P[A \cap (B \cup C)]}{P(B \cup C)} = \frac{P[(A \cap B) \cup (A \cap C)]}{P(B) + P(C) - P(B \cap C)} \\ &= \frac{P[A \cap B] + P[A \cap C] - P[A \cap B \cap C]}{P[B] + P[C]} \\ &= \frac{P[A] \cdot P[B] + P[A] \cdot P[C] - 0}{P[B] + P[C]} = P[A] \end{aligned}$$

E1)

$$\sum f_{x,y} = 1 \Rightarrow \frac{p}{2} + \frac{p}{4} + \frac{1}{4} + \frac{1}{2} = 1 \Rightarrow \frac{3}{4}p + \frac{3}{4} = 1 \Rightarrow p = \frac{1}{3}$$

$$\rho(5X, 3Y) = \rho_{X,Y} \text{ per le proprietà di } \rho.$$

$$\rho_{X,Y} = \frac{\text{cov}[X, Y]}{\sigma_X \cdot \sigma_Y}$$

$$E[XY] = -1 \cdot 3 \cdot \frac{1}{2} + 1 \cdot 1 \cdot \frac{1}{4} = -\frac{5}{4}$$

$$E[X] = 1 \cdot \frac{1}{4} + 3 \cdot \frac{1}{2} = \frac{7}{4}$$

$$E[Y] = -1 \cdot \left(\frac{1}{6} + \frac{1}{2}\right) + 1 \cdot \left(\frac{1}{12} + \frac{1}{4}\right) = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$$

$$\Rightarrow \text{cov}[X, Y] = -\frac{5}{4} + \frac{7}{4} \cdot \frac{1}{3} = \frac{7-15}{12} = -\frac{2}{3}$$

$$E[X^2] = 1 \cdot \frac{1}{4} + 9 \cdot \frac{1}{2} = \frac{19}{4}$$

$$E[Y^2] = 1 \cdot \frac{2}{3} + 1 \cdot \frac{1}{3} = 1$$

$$\Rightarrow \sigma_x^2 = \frac{19}{4} - \frac{49}{16} = \frac{27}{16}$$

$$\sigma_y^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow \rho = \frac{-2/3}{(3\sqrt{3}/4) \cdot (2\sqrt{2}/3)} = -\frac{2}{3} \cdot \frac{2}{\sqrt{6}} = \underline{\underline{-\frac{2}{9}\sqrt{6}}}$$

E2)

$M'_2 = \mu'_2$ dove $M'_2 = \bar{X}_m$ e $\mu'_2 = E[X]$ da calcolare

$$E[X] = \int_2^{+\infty} x \cdot \frac{3}{\theta} e^{-\frac{3}{\theta}(x-2)} dx$$

$$x-2 = y \quad x = y+2 \quad dx = dy$$

$$x=2 \rightarrow y=0$$

$$x=+\infty \rightarrow y=+\infty$$

$$\begin{aligned} E[X] &= \int_0^{+\infty} (y+2) \frac{3}{\theta} e^{-\frac{3}{\theta}y} dy \\ &= \int_0^{\infty} \left(-\frac{3}{\theta}\right)(y) e^{-\frac{3}{\theta}y} dy + (-)2 \int_0^{\infty} \left(-\frac{3}{\theta}\right) e^{-\frac{3}{\theta}y} dy \\ &= \left| (-y) e^{-\frac{3}{\theta}y} \right|_0^{+\infty} + \int_0^{+\infty} e^{-\frac{3}{\theta}y} dy - 2 \left| e^{-\frac{3}{\theta}y} \right|_0^{\infty} \end{aligned}$$

$$= \lim_{y \rightarrow +\infty} \frac{-y}{e^{\frac{3}{\theta}y}} - \frac{\theta}{3} \left[e^{-\frac{3}{\theta}y} \right]_0^{\infty} - 2 \left[e^{-\frac{3}{\theta}y} \right]_0^{\infty}$$

$$\stackrel{H}{=} \lim_{y \rightarrow +\infty} \frac{-1}{\frac{\theta}{3} e^{\frac{3}{\theta}y}} + \frac{\theta}{3} + 2 = \frac{\theta}{3} + 2$$

\downarrow
 0

Perciò $\bar{X}_m = \frac{\theta}{3} + 2 \Rightarrow \hat{\theta} = 3\bar{X}_m - 6$

$$\underline{E[\hat{\theta}] = E[3\bar{X}_m - 6] = 3E[\bar{X}_m] - 6 = 3\mu - 6 = 3\left(\frac{\theta}{3} + 2\right) - 6 = \theta}$$

$\hat{\theta}$ è corretto

$$MSE[T] = \text{var}[T] + \mathcal{D}^2(T)$$

$$\text{var}[3\bar{X}_m - 6] = \text{var}[3\bar{X}_m] = 9 \text{var}[\bar{X}_m] = 9 \frac{\sigma^2}{n}$$

$$\sigma^2 = \text{var}[x] = E[x^2] - E[x]^2$$

$$E[x^2] = \int_2^{+\infty} x^2 \cdot \frac{3}{\theta} e^{-\frac{3}{\theta}(x-2)} dx = \int_0^{\infty} (y+2)^2 \frac{3}{\theta} e^{-\frac{3}{\theta}y} dy$$

$$= \dots = \frac{2\theta^2}{9} + \frac{4}{3}\theta + 4$$

$$\Rightarrow \sigma^2 = \frac{2\theta^2}{9} + \frac{4}{3}\theta + 4 - \left(\frac{\theta}{3} + 2\right)^2 = \frac{\theta^2}{9}$$

$$\Rightarrow \underline{MSE[\hat{\theta}] = \frac{\theta^2}{3}}$$

$$\lim_{n \rightarrow \infty} MSE(\hat{\theta}) = 0 \Rightarrow \underline{\hat{\theta} \text{ è consistente}}$$

Prova scritta del 03.09.2008 (1a 1)

c1) $X \sim N(\mu=6.3, \sigma^2=4)$

$$\begin{aligned} \underline{P[\sqrt[3]{X} \geq 2]} &= P[X \geq 2^3] = P\left[Z > \frac{8-6.3}{2}\right] = P[Z > 0.85] \\ &= 1 - P[Z \leq 0.85] = 1 - \underset{\substack{\uparrow \\ \text{TAB}}}{0.80234} = \underline{0.19766} \end{aligned}$$

c2) 3R, 4B

$X \sim \text{Geom}(p)$ $f_x = p q^{x-1}$ dove $q = 1-p$

$P[X=3] = p(1-p)^2$ p è da calcolare

$$p = \frac{\binom{R}{1} \binom{B}{1}}{\binom{R+B}{2}} = \frac{\binom{3}{1} \cdot \binom{4}{1}}{\binom{7}{2}} = \frac{12}{21} = \frac{4}{7}$$

$$\Rightarrow \underline{P[X=3] = \frac{4}{7} \cdot \frac{9}{49} = \frac{36}{343}}$$

c3) $X \sim \text{exp}(\lambda)$ $E[X] = 400$

poiché $E[X] = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{400}$

$$\begin{aligned} P[X \geq 100 \mid X=500] &= \underset{\substack{\uparrow \\ \text{memoria}}}{P[X \geq 100]} = 1 - P[X < 100] \\ &= 1 - F(100) = 1 - [1 - e^{-\frac{1}{400} \cdot 100}] \\ &= e^{-1/4} = \frac{1}{\sqrt{e}} = 0.7788 \end{aligned}$$

Ricorda che se $f_x = \lambda e^{-\lambda x} \rightarrow F(x) = 1 - e^{-\lambda x}$

C4)

$$X \sim \text{Poisson} = P(\lambda) \quad f_x = \frac{\lambda^x}{x!} e^{-\lambda}$$

$$\alpha = 0.03$$

$$\lambda = 5\alpha = 5 \cdot 0.03 = 0.15$$

$$\underline{P[X=0]} = e^{-0.15} = \underline{0.86071}$$

QT) X, Y indip $\Rightarrow E[XY] = E[X] \cdot E[Y] \Rightarrow \text{cov}[X, Y] = 0$

$$\underline{\text{cov}[X-2Y, X+3]} = \text{cov}[X, X] + \text{cov}[X, 3] + \text{cov}[-2Y, X]$$
$$- \text{cov}[-2Y, 3]$$

$$= \text{cov}[X, X] - 2 \text{cov}[Y, X] = \underline{\text{var}[X]}$$

E1)

$$\mu_1' = \mu_1' \quad \mu_2' = \bar{X}_m \quad \mu_1' = \mu = E[X]$$

$$E[X] = \int_0^{+\infty} x \cdot \frac{16}{\theta^2} \cdot x \cdot e^{-\frac{4}{\theta}x} dx = \left(\frac{4}{\theta}\right) \int_0^{\infty} x^2 \left(\frac{4}{\theta} e^{-\frac{4}{\theta}x}\right) dx$$

$$= \frac{4}{\theta} \int_0^{\infty} y^2 \left(\frac{4}{\theta} e^{-\frac{4}{\theta}y}\right) dy \quad \text{posto } Y \sim \exp(\lambda = \frac{4}{\theta})$$

$$= \frac{4}{\theta} E[Y^2]$$

$$\text{ma } E[Y^2] = \text{var}[Y] + E[Y]^2$$

$$\text{e se } Y \sim \exp \quad E[Y] = \frac{1}{\lambda}, \quad \text{var}[Y] = \frac{1}{\lambda^2}$$

$$\text{quindi } E[Y^2] = \frac{2}{\lambda^2} = \frac{2}{\frac{16}{\theta^2}} = \frac{\theta^2}{8}$$

$$\Rightarrow E[X] = \frac{4}{\theta} \cdot \frac{\theta^2}{8} = \frac{\theta}{2}$$

oppure si procede col calcolo diretto dell'integrale iniziale.

$$\text{Pertanto } \bar{X}_m = \frac{\theta}{2} \Rightarrow \underline{\hat{\theta} = 2\bar{X}_m}$$

$$\underline{E[\hat{\theta}]} = E[2\bar{X}_m] = 2E[\bar{X}_m] = 2\mu = 2E[X] = 2 \cdot \frac{0}{2} = \underline{0}$$

$\hat{\theta}$ è corretto

$$T_2 = \frac{3X_1 + \bar{X}_m}{8}$$

$$\begin{aligned} E[T_2] &= \frac{1}{8} E[3X_1 + \bar{X}_m] = \frac{1}{8} \left\{ 3E[X_1] + E[\bar{X}_m] \right\} = \frac{1}{8} \cdot 4E[X] \\ &= \frac{1}{2} E[X] = \underline{\frac{0}{4}} \end{aligned}$$

$\Rightarrow T_2$ non è corretto.

$\Rightarrow \hat{\theta}$ è preferibile

E2)

$$F(x) = \int_{-\infty}^x f(t) dt$$

se $x \leq 0$ $F(x) = 0$

se $0 < x \leq 1$ $F(x) = \int_{-\infty}^0 f(t) dt + \int_0^x \frac{1}{2} t^2 dt = \frac{1}{2} \cdot \frac{1}{3} x^3 = \frac{1}{6} x^3$

se $1 < x \leq 2$ $F(x) = \int_{-\infty}^0 f(t) dt + \int_0^1 \frac{1}{2} t^2 dt + \int_1^x \frac{5}{6} dt$

$$= \frac{1}{2} \cdot \frac{1}{3} + \frac{5}{6} (x-1) = \frac{5}{6} x - \frac{2}{3}$$

se $x > 2$ $F(x) = 1$

quindi

$$F(x) = \begin{cases} 0 & \text{se } x \leq 0 \\ \frac{1}{6} x^3 & \text{se } 0 < x \leq 1 \\ \frac{5}{6} x - \frac{2}{3} & \text{se } 1 < x \leq 2 \\ 1 & \text{se } x > 2 \end{cases}$$

$$\text{var}[X] = E[X^2] - E[X]^2$$

$$E[X^2] = \int_{-6}^{+6} x^2 f(x) dx = \int_0^1 x^2 \cdot \frac{1}{2} x^2 dx + \int_1^2 \frac{5}{6} x^2 dx$$

$$= \frac{1}{2} \cdot \frac{1}{5} + \frac{5}{6} \cdot \frac{1}{3} (8-1) = \frac{1}{10} + \frac{35}{18} = \frac{22}{45}$$

$$E[X] = \int_0^1 x \cdot \frac{1}{2} x^2 dx + \int_1^2 \frac{5}{6} x dx$$

$$= \frac{1}{2} \cdot \frac{1}{4} + \frac{5}{6} \cdot \frac{1}{2} (4-1) = \frac{1}{8} + \frac{5}{4} = \frac{11}{8}$$

$$\Rightarrow \text{var}[X] = 443/2880$$

$$P\left[\frac{1}{2} \leq X \leq 4\right] = P\left[\frac{1}{2} \leq X \leq 2\right] = F(2) - F\left(\frac{1}{2}\right)$$

$$= 1 - \frac{1}{6} \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{48} = \frac{47}{48}$$

se $Y \sim \text{Unif.}[-1, 1]$ $E[Y] = \frac{1-1}{2} \equiv 0$ ($E[Y] = \frac{a+b}{2}$)

$$E[(X+Y)^2] = E[X^2 + 2XY + Y^2] = E[X^2] + 2E[XY] + E[Y^2]$$

$$= E[X^2] + 2E[X] \cdot E[Y] + E[Y^2]$$

\uparrow INDIP. \equiv \equiv 0

$$= E[X^2] + E[Y^2]$$

$$\text{var}[Y] = \frac{1}{3} \quad \left(\text{var}[Y] = \frac{(b-a)^2}{12}\right)$$

$$\text{var}[Y] = E[Y^2] - E[Y]^2 = E[Y^2]$$

\equiv
 0

$$\Rightarrow E[(X+Y)^2] = \frac{22}{45} + \frac{1}{3} = \frac{107}{45}$$

Prova scritta del 10.12.2008 (file 1)

C1) $X \sim N(\mu, \sigma^2)$ $\sigma^2: P[|X-\mu| > 2] = 0.1141$

$$\begin{aligned} P[|X-\mu| > 2] &= 1 - P[|X-\mu| \leq 2] = 1 - P[-2 \leq X-\mu \leq 2] \\ &= 1 - P\left[-\frac{2}{\sigma} \leq Z \leq \frac{2}{\sigma}\right] = 1 - 2P\left[0 \leq Z \leq \frac{2}{\sigma}\right] \\ &= 1 - 2\left\{P\left[Z \leq \frac{2}{\sigma}\right] - \frac{1}{2}\right\} = 2 - 2P\left[Z \leq \frac{2}{\sigma}\right] \end{aligned}$$

$$\Rightarrow P\left[Z \leq \frac{2}{\sigma}\right] = \frac{2 - 0.1141}{2} = 0.94295 \xrightarrow{\text{TAB}} z_{\alpha} = 1.58$$

$$\Rightarrow \frac{2}{\sigma} = 1.58 \Rightarrow \sigma = \frac{2}{1.58} = 1.2658 \Rightarrow \underline{\sigma^2 = 1.60230}$$

C2) $n = 100$ $p = 0.01$ guasto

$$X \sim B(100, 0.01)$$

$$P[X \geq 3] = 1 - P[X < 3] = 1 - P[X \leq 2] = 1 - P[X=0] - P[X=1] - P[X=2]$$

$$P[X=0] = \binom{100}{0} (0.01)^0 \cdot (0.99)^{100}$$

"1"

$$P[X=1] = \binom{100}{1} (0.01)^1 \cdot (0.99)^{99}$$

"100"

$$P[X=2] = \binom{100}{2} (0.01)^2 \cdot (0.99)^{98}$$

= 5050

sostituendo $P[X \geq 3] = 0.079373$

C3)

$$V \sim N(\mu=2, \sigma=1) \quad W = 5V^2$$

$$\begin{aligned} \underline{E[W]} &= 5E[V^2] = 5(\text{var}[V] + E[V]^2) \\ &= 5(1 + 4) = \underline{25} \end{aligned}$$

C4) la funzione di densità di X NON È NOTA.

$$\mu_x = 80 \quad \sigma_x^2 = 25$$

$$m: P[|\bar{X}_m - 80| \leq 5] \geq 0.8$$

Per la legge dei grandi numeri

$$\varepsilon = 5$$

$$1 - \delta = 0.8 \Rightarrow \delta = 0.2$$

$$m \geq \frac{\sigma^2}{\delta \varepsilon^2} = \frac{25}{25 \cdot 0.2} = \underline{5}$$

Q7)

$$\text{var} \left[\frac{X}{\sigma_x} - \frac{Y}{\sigma_y} \right] \geq 0 \Rightarrow \rho_{X,Y} \leq 1$$

$$\rho_{X,Y} = \frac{\text{cov}[X,Y]}{\sigma_x \cdot \sigma_y} \quad \text{perciò } \rho_{X,Y} \leq 1 \sim \text{cov}[X,Y] \leq \sigma_x \cdot \sigma_y$$

$$\begin{aligned} 0 \leq \text{var} \left[\frac{X}{\sigma_x} - \frac{Y}{\sigma_y} \right] &= \text{var} \left[\frac{X}{\sigma_x} \right] + \text{var} \left[\frac{Y}{\sigma_y} \right] - 2 \text{cov} \left[\frac{X}{\sigma_x}, \frac{Y}{\sigma_y} \right] \\ &= \frac{1}{\sigma_x^2} \cdot \sigma_x^2 + \frac{1}{\sigma_y^2} \cdot \sigma_y^2 - 2 \cdot \frac{1}{\sigma_x \cdot \sigma_y} \text{cov}[X,Y] \\ &= 2 - \frac{2}{\sigma_x \sigma_y} \text{cov}[X,Y] \end{aligned}$$

allora

$$-\frac{2}{\sigma_x \sigma_y} \text{cov}[X,Y] + 2 \geq 0 \quad \text{cioè } \text{cov}[X,Y] \leq \sigma_x \cdot \sigma_y$$

e quindi $\rho_{X,Y} \leq 1$.

E1)

$$\int_{-\infty}^{+\infty} f(x) dx = 1$$

$$1 = \int_{\frac{1}{2}}^{+\infty} \frac{k}{x^4} dx = k \cdot \left(-\frac{1}{3}\right) \left[x^{-3}\right]_{\frac{1}{2}}^{+\infty} = -\frac{k}{3} \cdot \left(-\frac{1}{\frac{1}{8}}\right)$$

$$= \frac{8}{3} \frac{k}{8} \Rightarrow \underline{\underline{k = \frac{3}{8} \cdot 8^3}}$$

$$M_1' = \mu_1' \quad \text{dove } M_1' = \bar{X}_m \quad \text{e } \mu_1' = \mu = E[X]$$

$$E[X] = \int_{-\frac{\theta}{2}}^{+\infty} x \cdot \frac{3}{8} \theta^3 \cdot \frac{1}{x^4} dx = \frac{3}{8} \theta^3 \cdot \left(-\frac{1}{2} \cdot \frac{1}{x^2} \right)_{-\frac{\theta}{2}}^{+\infty}$$

$$= \frac{3}{16} \theta^3 \cdot \frac{4}{\theta^2} = \frac{3}{4} \theta$$

$$\Rightarrow \bar{X}_m = \frac{3}{4} \theta \quad \text{dove } \hat{\theta} = \frac{4}{3} \bar{X}_m$$

$$\underline{E[\hat{\theta}]} = E\left[\frac{4}{3} \bar{X}_m\right] = \frac{4}{3} E[\bar{X}_m] = \frac{4}{3} \mu = \frac{4}{3} E[X] = \frac{4}{3} \cdot \frac{3}{4} \theta = \underline{\theta}$$

$\hat{\theta}$ è corretto

$$MSE[\hat{\theta}] = \text{var}[\hat{\theta}] + \overset{=0}{\theta^2[\hat{\theta}]} = \text{var}\left[\frac{4}{3} \bar{X}_m\right] = \frac{16}{9} \text{var}[\bar{X}_m]$$

$$= \frac{16}{9} \frac{\sigma^2}{n}$$

$$\sigma^2 = E[X^2] - E[X]^2$$

$$E[X^2] = \int_{-\frac{\theta}{2}}^{+\infty} x^2 \cdot \frac{3}{8} \theta^3 \cdot \frac{1}{x^4} dx = \frac{3}{8} \theta^3 \cdot \left(-\frac{1}{2} \right)_{-\frac{\theta}{2}}^{+\infty} = \frac{3}{8} \theta^3 \cdot \frac{4}{\theta^2} = \frac{3}{4} \theta^2$$

$$\sigma^2 = \frac{3}{4} \theta^2 - \frac{9}{16} \theta^2 = \frac{3}{16} \theta^2$$

$$\Rightarrow \underline{MSE[\hat{\theta}]} = \frac{16}{9} \cdot \frac{3}{16} \frac{\theta^2}{n} = \underline{\frac{\theta^2}{3n}}$$

E2)

$$\text{se } X = -1 \quad Y = 2$$

$$f_x(-1) = \frac{1}{2}$$

$$\text{se } X = 0 \quad Y = 1$$

$$f_x(0) = \frac{1}{3}$$

$$\text{se } X = 1 \quad Y = 2$$

$$f_x(1) = \frac{1}{6}$$

} sono le marginali della X.

costruisco la tabella a doppia entrata

	X=-1	X=0	X=1	f_Y
Y=1	○	●	○	
Y=2	●	○	●	
f_X	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	①

Si ricava subito $f_{X,Y}(-1,1) = f_{X,Y}(0,2) = f_{X,Y}(1,1) = 0$

e che $f_{X,Y}(-1,2) = \frac{1}{2}$

$f_{X,Y}(0,1) = \frac{1}{3}$

$f_{X,Y}(1,2) = \frac{1}{6}$

pertanto $f_Y(1) = \frac{1}{3}$ e $f_Y(2) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ (o subito da $1 - \frac{1}{3}$)

X e Y sono dipendenti

$$f_{X,Y}(-1,2) \neq f_X(-1) \cdot f_Y(2)$$

$$\begin{matrix} \parallel & \parallel & \parallel \\ 0 & \frac{1}{2} & \frac{2}{3} \end{matrix}$$

$$\rho_{X,Y} = \frac{E[XY] - E[X] \cdot E[Y]}{\sigma_X \cdot \sigma_Y}$$

$$E[X] = -1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{6} = -\frac{1}{3}$$

$$E[Y] = 1 \cdot \frac{1}{3} + 2 \cdot \frac{2}{3} = \frac{5}{3}$$

$$E[XY] = (-1) \cdot 2 \cdot \frac{1}{2} + 1 \cdot 2 \cdot \frac{1}{6} = -\frac{2}{3}$$

$$E[X^2] = 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{6} = \frac{2}{3} \Rightarrow \sigma_X^2 = \frac{2}{3} - \frac{1}{9} = \frac{5}{9} \Rightarrow \sigma_X = \frac{\sqrt{5}}{3}$$

$$E[Y^2] = 1 \cdot \frac{1}{3} + 4 \cdot \frac{2}{3} = 3 \Rightarrow \sigma_Y^2 = 3 - \frac{25}{9} = \frac{2}{9} \Rightarrow \sigma_Y = \frac{\sqrt{2}}{3}$$

$$\Rightarrow \rho_{X,Y} = \left(-\frac{2}{3} + \frac{5}{9}\right) / \left(\frac{\sqrt{10}}{9}\right) = -\frac{1}{\sqrt{10}}$$